

A new class of f -deformed charge coherent states and their nonclassical properties

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March 2, 2013

Abstract

Two-mode charge (pair) coherent states has been introduced previously by using $\langle \eta |$ representation. In the present paper we reobtain these states by a rather different method. Then, using the nonlinear coherent states approach and based on a simple manner by which the representation of two-mode charge coherent states is introduced, we generalize the bosonic creation and annihilation operators to the f -deformed ladder operators and construct a new class of f -deformed charge coherent states. Unlike the (linear) pair coherent states, our presented structure has the potentiality to generate a large class of pair coherent states with various nonclassicality signs and physical properties which are of interest. Along this purpose, we use a few well-known nonlinearity functions associated with particular quantum systems as some physical appearances of our presented formalism. After introducing the explicit form of the above correlated states in two-mode Fock-space, several nonclassicality features of the corresponding states (as well as the two-mode linear charge coherent states) are numerically investigated by calculating quadrature squeezing, Mandel parameter, second-order correlation function, second-order correlation function between the two modes and Cauchy-Schwartz inequality. Also, the oscillatory behaviour of the photon count and the quasi-probability (Husimi) function of the associated states will be discussed.

Pacs: 42.50.Dv, 42.50.-p

Keywords: Nonlinear coherent state, Two-mode charge coherent states, f -deformed charge coherent states, Charge operator, Nonclassical states.

1 Introduction

Coherent states are venerable objects in many areas of physical researches in recent decades [1, 2], with special place in the quantum optics [3, 4]. Therefore, various generalizations have been proposed up to now. In this paper, we confine ourselves to two-mode type of generalization of coherent states. Along this latter subject, we start with a brief review on the charge (pair) coherent states. Horn and Silver [5] defined the so-called charge coherent state $|\alpha, q\rangle$, which is the common eigenvector of number difference operator, sometimes has been named charge operator, given by

$$\hat{Q} = \hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b} \quad (1)$$

and pair annihilation operator $\hat{a}\hat{b}$ with eigenvalues q and α , respectively, i.e.,

$$\hat{Q}|\alpha, q\rangle = q|\alpha, q\rangle, \quad (2)$$

$$\hat{a}\hat{b}|\alpha, q\rangle = \alpha|\alpha, q\rangle, \quad (3)$$

where $\hat{a}(\hat{a}^\dagger)$ and $\hat{b}(\hat{b}^\dagger)$ are bosonic annihilation (creation) operators and q is an integer has been named "charge number" which is indeed the photon number difference between the two modes of the field. These states, sometimes have been called pair coherent states, have been used for the description of the production of pions [6] and other problems in quantum field theory [1]. The explicit form of $|\alpha, q\rangle$, represents a well-known class of states within the general theory of coherent states, reads as

$$|\alpha, q\rangle = N_q(|\alpha|^2)^{-1/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n![n+|q|]!}} \left| n + \frac{q+|q|}{2}, n - \frac{q-|q|}{2} \right\rangle, \quad (4)$$

where $\alpha \in \mathbb{C}$, the kets $|m, n\rangle$ are the two-mode number states and N_q is an appropriate normalization factor may be determined. The states in (4) include two distinct sets of charge coherent states corresponding to $q \geq 0$ and $q \leq 0$. In (4) the following definition has been assumed

$$[n + |q|]! \doteq (1 + |q|)(2 + |q|)\dots(n + |q|). \quad (5)$$

An experimental scheme for generation of the states in (4) has been proposed by Agarwal [7, 8]. Recently, by using the nonlinear coherent states method [9, 10] one of us with his co-author have introduced the nonlinear charge coherent states in a general structure which will be briefly discussed here [11]. Consider f -deformed ladder operators

$$\begin{aligned} \hat{A} &= \hat{a}f(\hat{n}_a), & \hat{A}^\dagger &= f^\dagger(\hat{n}_a)\hat{a}^\dagger, \\ \hat{B} &= \hat{b}f(\hat{n}_b), & \hat{B}^\dagger &= f^\dagger(\hat{n}_b)\hat{b}^\dagger, \end{aligned} \quad (6)$$

where $\hat{a}(\hat{b})$, $\hat{a}^\dagger(\hat{b}^\dagger)$ and $\hat{n}_a = \hat{a}^\dagger\hat{a}$ ($\hat{n}_b = \hat{b}^\dagger\hat{b}$), are respectively bosonic annihilation, creation and number operators of mode $a(b)$, and $f(n)$ is an operator-valued function of intensity of radiation field (from now on has been assumed to be real) characterizes the nonlinearity nature of physical systems. The pair f -deformed annihilation operator $\hat{A}\hat{B}$ commutes with the charge operator, i.e., $[\hat{Q}, \hat{A}\hat{B}] = 0$. Thus, the latter two operators should satisfy the following eigenvalue equations

$$\hat{Q}|\xi, q, f\rangle = q|\xi, q, f\rangle, \quad (7)$$

$$\hat{A}\hat{B}|\xi, q, f\rangle = \xi|\xi, q, f\rangle, \quad \xi \in \mathbb{C}, \quad (8)$$

where \hat{Q} and q keep their previous definitions expressed in (1). These eigenstates have in general two distinct representations which can be put in a single expression as follows:

$$|\xi, q, f\rangle = N(|\xi|^2)^{-1/2} \sum_{n=0}^{\infty} \frac{\xi^n}{\sqrt{n![n+q]!}[f(n)]![f(n+q)]!} \left| n + \frac{q+|q|}{2}, n - \frac{q-|q|}{2} \right\rangle, \quad (9)$$

with the normalization constant given by

$$N(|\xi|^2) = \sum_{n=0}^{\infty} \frac{|\xi|^{2n}}{n![n+|q|]!([f(n)]![f(n+|q|)]!)^2}. \quad (10)$$

Note that, in obtaining (9) and (10) we have used the conventional definitions

$$f(n)]! \doteq f(n)f(n-1)f(n-2)\cdots f(1), \quad [f(0)]! \doteq 1, \quad (11)$$

and

$$\begin{aligned} [f(n+|q|)]! &\doteq f(n+|q|)f(n-1+|q|)f(n-2+|q|)\cdots f(1+|q|), \\ [f(|q|)]! &\doteq 1. \end{aligned} \quad (12)$$

On the other hand, Fan et al [12] introduced $|q, \lambda\rangle$ [13], i.e., the common eigenstate of \hat{Q} defined in (1) and (Hermitian) \hat{g} operator is given by

$$\hat{g} = (\hat{a} + \hat{b}^\dagger)(\hat{a}^\dagger + \hat{b}), \quad (13)$$

with eigenvalues q and λ , respectively, i.e.,

$$\hat{Q}|q, \lambda\rangle = q|q, \lambda\rangle, \quad (14)$$

$$\hat{g}|q, \lambda\rangle = \lambda|q, \lambda\rangle, \quad \lambda \geq 0, \quad (15)$$

by using $\langle\eta|$ representation. This is due to the fact that $[\hat{Q}, \hat{g}] = 0$. The explicit form of $|q, \lambda\rangle$ in two-mode Fock-space, has been deduced by the authors reads as

$$|q, \lambda\rangle = e^{-\frac{\lambda}{2}} \sum_{n=\max(0, -q)}^{\infty} H_{n+q, n}(\sqrt{\lambda}, \sqrt{\lambda}) \frac{1}{\sqrt{(n+q)!n!}} |n+q, n\rangle, \quad (16)$$

where $H_{m, n}$ is the two-variable Hermite polynomial, has been defined as

$$H_{m, n}(z, z^*) = \sum_{k=0}^{\min(m, n)} \frac{(-1)^k m! n!}{k!(m-k)!(n-k)!} z^{m-k} z^{*n-k}. \quad (17)$$

In the present paper, our main aims may be expressed as follows: i) reobtaining the explicit form of $|q, \lambda\rangle$ in two-mode Fock-space by using a rather different method other than the $\langle\eta|$ representation has been followed in [12], ii) generalizing \hat{g} operator (combination of bosonic annihilation and creation operators in (13)) to \hat{G} operator (combination of f -deformed ladder operators) and obtaining the common eigenstates of \hat{Q} and \hat{G} have been called

by us as " f -deformed charge coherent states"² and finally iii) investigating some of the nonclassical features and quantum statistical properties of the f -deformed charge coherent states associated with a few quantum systems with particular nonlinearity functions, in addition to the state $|q, \lambda\rangle$ which is indeed a special case of our f -deformed charge coherent states with $f(n) = 1$. Obviously, our new type of f -deformed charge coherent states in the present paper is substantially different from the states in (9) have been introduced in [11].

The paper is organized as follows. In Sec. 2, we will reobtain the two-mode (linear) charge coherent states, which is the common eigenvector of \hat{Q} and \hat{g} operators. Then, in Sec. 3, we will generalize the creation and annihilation operators to the f -deformed operators and construct the f -deformed charge coherent states. Next, in Sec. 4, as some physical realizations of the formalism we consider a few particular nonlinearity functions and then we study some of the nonclassical features and quantum statistical properties of the two-mode (linear) charge coherent states and f -deformed charge coherent states associated with those physical systems. At last, in Sec. 5, we will present a summery and conclusion.

2 Two-mode (linear) charge coherent state: common eigenstate of \hat{Q} and \hat{g} operators

Two-mode (linear) charge coherent state are common eigenstate of charge operator introduced in (1) and \hat{g} operator defined in (13) respectively with eigenvalues q and ξ have been expressed in (14) and (15). It is worthwhile noticing that we start our discussion by imposing a little modification in the notation of Ref. [12] by changing the eigenstate $|q, \lambda\rangle$ to $|\xi, q\rangle$. The form of the state in two-mode Fock-space is considered to be

$$|\xi, q\rangle = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} c_{n,m} |n, m\rangle. \quad (18)$$

Substituting (18) in (14), one readily finds

$$n = m + q. \quad (19)$$

²We have selected this name to be distinguished from the previous nonlinear charge coherent states in [11], whereas it is clear that both classes of states are in fact nonlinear or f -deformed.

With the latter result in mind, the explicit form of the state may be rewritten as

$$|\xi, q\rangle^{(+)} = \sum_{n=0}^{\infty} c_{n+q,n}^{(+)} |n+q, n\rangle, \quad q \geq 0, \quad (20)$$

$$|\xi, q\rangle^{(-)} = \sum_{n=0}^{\infty} c_{n,n-q}^{(-)} |n, n-q\rangle, \quad q \leq 0. \quad (21)$$

Now by substituting (20), for instance, into (15) we find the recursion relation

$$\begin{aligned} c_{n+q,n}^{(+)} &= \frac{1}{\sqrt{(n+q)n}} \{c_{n+q-1,n-1}^{(+)} [\xi - (n+q) - (n-1)] \\ &- c_{n+q-2,n-2}^{(+)} \sqrt{(n+q-1)(n-1)}\}. \end{aligned} \quad (22)$$

$$(23)$$

By straightforward but lengthy procedure the expansion coefficients are then obtained in terms of $c_{q,0}^{+}$ as follows

$$c_{n+q,n}^{(+)} = c_{q,0}^{(+)} \sqrt{[n+q]!n!} \sum_{k=0}^n \frac{(-1)^k \xi^{n-k}}{k![n+q-k]!(n-k)!}, \quad (24)$$

and thus the explicit form of the state for $q \geq 0$ may be expressed as

$$|\xi, q\rangle^{(+)} = c_{q,0}^{(+)} \sum_{n=0}^{\infty} \sqrt{[n+q]!n!} \sum_{k=0}^n \frac{(-1)^k \xi^{n-k}}{k![n+q-k]!(n-k)!} |n+q, n\rangle. \quad (25)$$

Similar procedure can be performed for $q \leq 0$, which led us to the explicit form of (linear) charge coherent state for $q \leq 0$ as

$$|\xi, q\rangle^{(-)} = c_{q,0}^{(-)} \sum_{n=0}^{\infty} \sqrt{n![n-q]!} \sum_{k=0}^n \frac{(-1)^k \xi^{n-k}}{k![n-q-k]!(n-k)!} |n, n-q\rangle. \quad (26)$$

In (25) and (26) $\xi \in \mathbb{C}$, and the normalization constants can be easily calculated from:

$$c_{q,0}^{(\pm)} = \left[\sum_{n=0}^{\infty} [n+|q|]!n! \left(\sum_{k=0}^n \frac{(-1)^k \xi^{n-k}}{k![n+|q|-k]!(n-k)!} \right)^2 \right]^{-1/2}. \quad (27)$$

By setting $q \geq 0$ and $q \leq 0$, in (27) one obtains the exact form of normalization factors of states in (25) and (26), respectively. It is worth mentioning that in deriving the relations (24)-(27) we have used the following definition

$$\begin{aligned} [n + |q| - k]! &\doteq (1 + |q|)(2 + |q|) \cdots (n + |q| - k) \\ &= \frac{(n + |q| - k)!}{|q|!}, \end{aligned} \quad (27)$$

which is a slightly different from the conventional definition of $[f(n)]!$ frequently has been used in the literature devoted with f -deformed coherent states. Noticing that the state in (16) has been introduced in [12] is unnormalized, it is easy to check that our final states in (25) and (26) is in exact consistence with (16), when one sets $\xi = \xi^* = \sqrt{\lambda}$ in (25) and (26) with $\lambda \in \mathbb{R}$. Also, note that the domain of the states obtained by us is the entire space of complex plane, while in Ref. [12] λ is real and positive. We would like to mention that our states, were obtained by a simpler and at the same time more general manner, are normalized to unity, while the state introduced in (16) are not normalized. In addition to this fact, to the best of our knowledge, the statistical properties and nonclassical features of these states have not been yet discussed in the literature. Henceforth, we will pay attention to this subject in the continuation of the paper, too, since any generalization scheme without this discussion seems to be poor from physical points of view.

3 Introducing the f -deformed charge coherent states

In this section, using the nonlinear coherent states method and based on the procedure by which the charge coherent states are derived in the previews section, we firstly generalize the bosonic creation and annihilation operators to the f -deformed operators defined in (6) and therefore \hat{g} which has been defined in (13) will be converted to \hat{G} which is given by

$$\hat{G} = (\hat{A} + \hat{B}^\dagger)(\hat{A}^\dagger + \hat{B}). \quad (28)$$

Now, we introduce f -deformed charge coherent states as simultaneous eigenstates of charge operator in (1) and \hat{G} in (28) with eigenvalues q and ξ ,

respectively, i.e.,

$$\hat{Q}|\xi, q, f\rangle = q|\xi, q, f\rangle, \quad (29)$$

$$\hat{G}|\xi, q, f\rangle = \xi|\xi, q, f\rangle, \quad (30)$$

since it can be easily checked that $[\hat{Q}, \hat{G}] = 0$ ². The explicit form of the common eigenstate in two-mode Fock-space is described as

$$|\xi, q, f\rangle = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} c_{n,m} |n, m\rangle. \quad (31)$$

Substituting (31) in (29), one obtains the same expression as in (19). With this result in mind, the explicit form of the state can be rewritten as

$$|\xi, q, f\rangle^{(+)} = \sum_{n=0}^{\infty} c_{n+q,n}^{(+)} |n+q, n\rangle, \quad q \geq 0, \quad (32)$$

$$|\xi, q, f\rangle^{(-)} = \sum_{n=0}^{\infty} c_{n,n-q}^{(-)} |n, n-q\rangle, \quad q \leq 0, \quad (33)$$

where the superscript $+$ ($-$) indicates the positive (negative) values of q parameter. Now, substituting the state (32), for instance, into (30) one finds a complicated recursion relation for $q \geq 0$ as follows:

$$\begin{aligned} c_{n+q,n}^{(+)} &= \frac{1}{\sqrt{(n+q)n}f(n+q)f(n)} \\ &\times [c_{n+q-1,n-1}^{(+)}(\xi - (n+q)f^2(n+q) - (n-1)f^2(n-1)) \\ &- c_{n+q-2,n-2}^{(+)}(\sqrt{(n+q-1)(n-1)})f(n+q-1)f(n-1)]. \end{aligned} \quad (32)$$

However, by defining

$$B_n^{(+)} = \frac{c_{n+q,n}^{(+)}}{c_{n+q-1,n-1}^{(+)}} \quad (33)$$

$$B_{n-1}^{(+)} = \frac{c_{n+q-1,n-1}^{(+)}}{c_{n+q-2,n-2}^{(+)}} \quad (34)$$

²It is worth mentioning that we can also deform the bosonic charge Q to f -deformed charge operator, however it then will not commute with neither of the operators $\hat{a}\hat{b}$, $\hat{A}\hat{B}$, \hat{g} or \hat{G} . So, no common eigenstate may be expected.

the relation (34) can be inverted to the form

$$B_n^{(+)} = \frac{1}{\sqrt{(n+q)nf(n+q)f(n)}} \{ [\xi - (n+q)f^2(n+q) - (n-1)f^2(n-1)] - \frac{\sqrt{(n+q-1)(n-1)f(n+q-1)f(n-1)}}{B_{n-1}^{(+)}} \}. \quad (34)$$

From the above equation (35), we may obtain an explicit expression for $B_{n-1}^{(+)}$ in terms of $B_{n-2}^{(+)}$. Setting the obtained $B_{n-1}^{(+)}$ in (35) we arrive at $B_n^{(+)}$ in terms of $B_{n-2}^{(+)}$. Continuing this procedure we can obtain $B_n^{(+)}$ in terms of $B_{n-3}^{(+)}, \dots$. Finally, we arrive at the complicated expression for $B_n^{(+)}$ in terms of $B_0^{(+)}$ which is assumed to be 1:

$$B_n^{(+)} = \frac{1}{\sqrt{(n+q)nf(n+q)f(n)}} \left[[\xi - (n+q)f^2(n+q) - (n-1)f^2(n-1)] - \frac{(n+q-1)(n-1)f^2(n+q-1)f^2(n-1)}{[\xi - (n+q-1)f^2(n+q-1) - (n-2)f^2(n-2)] - \frac{(n+q-2)(n-2)f^2(n+q-2)f^2(n-2)}{\vdots}} \right] \frac{1}{[\xi - (1+q)f^2(1+q)]}. \quad (32)$$

Note that the following relations hold

$$\begin{aligned} c_{n+q,n}^{(+)} &= B_n^{(+)} c_{n+q-1,n-1}^{(+)} \\ &= B_n^{(+)} B_{n-1}^{(+)} c_{n+q-2,n-2}^{(+)} \\ &= B_n^{(+)} B_{n-1}^{(+)} B_{n-2}^{(+)} c_{n+q-3,n-3}^{(+)} \\ &\vdots \\ &= B_n^{(+)} B_{n-1}^{(+)} B_{n-2}^{(+)} \dots B_0^{(+)} c_{q,0}^{(+)}, \quad B_0^{(+)} \doteq 1, \end{aligned} \quad (29)$$

or in a compact form we have:

$$c_{n+q,n}^{(+)} = [B_n^{(+)}]! c_{q,0}^{(+)}. \quad (30)$$

Adding all of the above results, one readily deduces the explicit form of f -deformed charge coherent states for $q \geq 0$ as follows:

$$|\xi, q, f\rangle^{(+)} = c_{q,0}^{(+)} \sum_{n=0}^{\infty} [B_n^{(+)}]! |n+q, n\rangle. \quad (31)$$

The normalization constant in (31) is given by

$$c_{q,0}^{(+)} = \left(\sum_{n=0}^{\infty} |[B_n^{(+)}]!|^2 \right)^{-1/2}. \quad (32)$$

Similar procedure may be followed for $q \leq 0$, which leads one to the explicit form of f -deformed charge coherent states as

$$|\xi, q, f\rangle^{(-)} = c_{q,0}^{(-)} \sum_{n=0}^{\infty} [B_n^{(-)}]! |n, n-q\rangle, \quad (33)$$

where $B_n^{(-)}$ may be expressed as

$$B_n^{(-)} = \frac{1}{\sqrt{n(n-q)}f(n)f(n-q)} \{ [\xi - (n)f^2(n) - (n-q-1)f^2(n-q-1)] \\ - \frac{(n-1)(n-q-1)f^2(n-1)f^2(n-q-1)}{[(\xi - (n-1)f^2(n-1) - (n-q-2)f^2(n-q-2)) - \frac{(n-2)(n-q-2)f^2(n-2)f^2(n-q-2)}{\vdots} \\ [\xi - f^2(1) + (q)f^2(-q)]] \}. \quad (31)$$

The normalization constant in (33) can be simply obtained as:

$$c_{q,0}^{(-)} = \left(\sum_{n=0}^{\infty} |[B_n^{(-)}]!|^2 \right)^{-1/2}. \quad (32)$$

Notice that in all above formula we have replaced $B_n^{(\pm)}(\xi, q, f)$ with $B_n^{(\pm)}$ for simplicity. It can be easily checked that setting $f(n) = 1$ in (31) and (33) recover (25) and (26), respectively.

4 Physical properties of the introduced states

As is required, we briefly review some of the criteria which will be used in the continuation of the paper for investigating the nonclassicality of the introduced states, as Mandel parameter, second-order correlation function, second-order correlation function between the two modes, Cauchy-Schwartz inequality and finally the quasi-probability function $Q(\alpha)$.

4.1 Nonclassicality criteria

- Using the definitions of position and momentum quadratures as $x = (\hat{a} + \hat{a}^\dagger)/\sqrt{2}$ and $p = (\hat{a} - \hat{a}^\dagger)/i\sqrt{2}$, it is well-known that the squeezing respectively occurs in x or p if $(\Delta x)^2 < 0.5$ or $(\Delta p)^2 < 0.5$. For instance, for position quadrature one has

$$\begin{aligned} (\Delta x)^2 &= \langle x^2 \rangle - \langle x \rangle^2 \\ &= \frac{1}{2}[\langle \hat{a}^2 \rangle + \langle \hat{a}^{\dagger 2} \rangle + \langle \hat{a}^\dagger \hat{a} \rangle + \langle \hat{a} \hat{a}^\dagger \rangle - \langle \hat{a} \rangle^2 - \langle \hat{a}^\dagger \rangle^2 - 2\langle \hat{a} \rangle \langle \hat{a}^\dagger \rangle] \end{aligned} \quad (32)$$

By calculating the necessary mean values over the states $|\xi, q, f\rangle^{(+)}$ it is easily seen that $\langle \hat{a} \rangle = \langle \hat{a}^\dagger \rangle = \langle \hat{a}^2 \rangle = \langle \hat{a}^{\dagger 2} \rangle = 0$; All of these arise from the fact that $\langle m, n | m', n' \rangle = \delta_{m, m'} \delta_{n, n'}$. Therefore, $(\Delta x)^2 = 1/2[\langle \hat{a}^\dagger \hat{a} \rangle + \langle \hat{a} \hat{a}^\dagger \rangle] = \langle \hat{a}^\dagger \hat{a} \rangle + 1/2$; Similar calculations for momentum quadrature lead us to the same result for $(\Delta p)^2$. So, $(\Delta x)^2 = (\Delta p)^2$ and this, clearly, irrespective of the value of $\langle \hat{a}^\dagger \hat{a} \rangle$, immediately leads one to conclude that no quadrature squeezing may be expected. The same discussion can be followed for our states with $q < 0$.

- Commonly, photon-counting statistics of the quantum states is investigated by evaluating Mandel parameter has been defined as [14],

$$Q_i = \frac{\langle \hat{n}_i^2 \rangle - \langle \hat{n}_i \rangle^2}{\langle \hat{n}_i \rangle} - 1, \quad (32)$$

where i stands for the two modes a, b and so $\hat{n}_a = \hat{a}^\dagger \hat{a}$, $\hat{n}_b = \hat{b}^\dagger \hat{b}$. The states for which $Q_{a(b)} = 0$, $Q_{a(b)} < 0$ and $Q_{a(b)} > 0$, respectively corresponds to Poissonian (standard coherent states), sub-Poissonian (non-classical states) and super-Poissonian (classical states) statistics.

- Even though there are quantum states in which supper-/sub-Poissonian statistical behavior is appeared simultaneously with bunching/antibunching effect, but this is not absolutely true [15]. Therefore, to investigate bunching or antibunching effects, second-order correlation function, defined as [16]:

$$g_a^{(2)}(0) = \frac{\langle \hat{a}^{\dagger 2} \hat{a}^2 \rangle}{\langle \hat{a}^\dagger \hat{a} \rangle^2}, \quad (32)$$

is widely used. Depending on the particular nonlinearity function $f(n)$, has been chosen for the construction of any class of f -deformed charge

coherent states, $g^2(0) > 1$ and $g^2(0) < 1$ respectively indicates bunching and antibunching effects. The case $g^2(0) = 1$ corresponds particularly to the canonical coherent states.

- Second-order correlation function between the two modes of the radiation field has been defined as [17]

$$g_{12}^{(2)}(0) = \frac{\langle \hat{a}^\dagger \hat{a} \hat{b}^\dagger \hat{b} \rangle}{\langle \hat{a}^\dagger \hat{a} \rangle \langle \hat{b}^\dagger \hat{b} \rangle}. \quad (32)$$

This quantity shows that the two modes are correlated and the state is classical if $g_{12}^{(2)}(0) > 1$; otherwise it is nonclassical.

- As another quantity, recall that the Cauchy-Schwartz inequality [17] has been defined as

$$I_0 = \frac{[\langle \hat{a}^{\dagger 2} \hat{a}^2 \rangle \langle \hat{b}^{\dagger 2} \hat{b}^2 \rangle]^{1/2}}{|\langle \hat{a}^\dagger \hat{a} \hat{b}^\dagger \hat{b} \rangle|} - 1 < 0. \quad (32)$$

If this inequality is violated, then the state is nonclassical.

It is to be mentioned that each of the above signs are sufficient, not necessary, for a state to be nonclassical, so we pay not more attention to other nonclassicality criteria.

- Different quasi-probability functions have been proposed in quantum optics [18]. Among them we pay attention to the Husimi function $Q(\alpha)$ which is defined for the single mode quantum state $|\psi\rangle$ as $Q(\alpha) = |\langle \alpha | \psi \rangle|^2 / \pi$, where $|\alpha\rangle$ is the canonical coherent states. This positive definite function which is defined over the phase space can be constructed in the homodyne experiments [19]. By this function the quantum interference effects in phase space have been illustrated [20]. Seemingly, it is possible to generalize this definition to our two-mode introduced states as follows

$$Q(\alpha_1, \alpha_2, \xi) = \frac{1}{\pi} |\langle \alpha_1, \alpha_2 | \psi(\xi) \rangle|^2, \quad (33)$$

where $\langle \alpha_1, \alpha_2 |$ is the bra of the ket $|\alpha_1, \alpha_2\rangle$:

$$\begin{aligned} |\alpha_1, \alpha_2\rangle &= |\alpha_1\rangle \otimes |\alpha_2\rangle \\ &= e^{-\frac{|\alpha_1|^2}{2}} e^{-\frac{|\alpha_2|^2}{2}} \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \frac{\alpha_1^{n_1} \alpha_2^{n_2}}{\sqrt{n_1! n_2!}} |n_1, n_2\rangle. \end{aligned} \quad (33)$$

Therefore, one can plot this function by fixing α_1 or α_2 in three-dimensional graphs.

4.2 Some physical appearances of the introduced states

In this section we want to investigate the nonclassical properties of the reobtained two-mode (linear) charge coherent states and the f -deformed charge coherent states. Clearly, choosing different $f(n)$'s lead to a variety of states with various physical properties. Therefore, one can not investigate the physical properties of our introduced states in (31) and (33) unless the special physical system of interest is determined. In this work we confine ourselves to three types of well-known nonlinearity functions.

i) Penson-Solomon nonlinearity function is defined by $f_{PS}(n) = p^{1-n}$, where $0 \leq p \leq 1$. This function, indeed, may be executed from the special class of coherent states defined by Penson and Solomon [21] by using the nonlinear coherent states method [22].

ii) A nonlinearity of the type $f(n) = \sqrt{n}$. This function appears in a natural way in the Hamiltonian describing the interaction between a two-level atom and electromagnetic field with intensity dependent coupling [23, 24].

iii) The \mathbf{q} -deformed nonlinearity associated with the well-known \mathbf{q} -coherent states, defined as $f_{\mathbf{q}}(n) = \sqrt{\frac{\mathbf{q}^n - \mathbf{q}^{-n}}{n(\mathbf{q} - \mathbf{q}^{-1})}}$ [10]. Note that since charge is denoted usually by q , therefore we have used \mathbf{q} notation in defining the \mathbf{q} -nonlinearity function.

Inserting each of these three nonlinearity functions in (31) and (33) one readily may obtain the explicit form of the associated f -deformed charge coherent states.

- Evaluating the Mandel parameter for the f -deformed positive charge coherent states requires the following expectation values

$$\begin{aligned} \langle \hat{n}_a \rangle^{(+)} &= {}^{(+)}\langle \xi, q, f | \hat{n}_a | \xi, q, f \rangle^{(+)} \\ &= |c_{q,0}^{(+)}|^2 \sum_{n=0}^{\infty} |[B_n^{(+)}]|^2 (n+q), \end{aligned} \quad (33)$$

and

$$\begin{aligned}
\langle \hat{n}_a^2 \rangle^{(+)} &= {}^{(+)}\langle \xi, q, f | \hat{n}_a^2 | \xi, q, f \rangle^{(+)} \\
&= |c_{q,0}^{(+)}|^2 \sum_{n=0}^{\infty} |[B_n^{(+)}]|^2 (n+q)^2,
\end{aligned} \tag{33}$$

where $\hat{n}_a = \hat{a}^\dagger \hat{a}$. In Fig. 1, the Mandel parameter for f -deformed charge coherent states is shown for Penson-Solomon nonlinearity function with $p = 0.5$ and fixed charge $q = 1$, as well as the \mathbf{q} -deformation function with $\mathbf{q} = 7$ and fixed charge $q = 2$. It is seen that, for both cases, this parameter is always negative, and so the sub-Poissonian behavior is visible. Also, note that its value is nearly $\simeq -1$, indicates the high strength nonclassicality of the considered states. It ought to be mentioned that we have also calculated the Mandel parameter for $f(n) = \sqrt{n}$ and $f(n) = 1$, but in both of the latter cases it gets very high positive value relative to the previous ones, such that displaying them in this figure would make the graphs unclear. Moreover, we were sure about the lack of this nonclassicality criteria for the other two functions.

- To calculate the second-order correlation function for mode a , defined in (4.1), the following relation is needed:

$$\begin{aligned}
\langle \hat{a}^{\dagger 2} \hat{a}^2 \rangle^{(+)} &= {}^{(+)}\langle \xi, q, f | \hat{a}^{\dagger 2} \hat{a}^2 | \xi, q, f \rangle^{(+)} \\
&= |c_{q,0}^{(+)}|^2 \sum_{n=0}^{\infty} |[B_n^{(+)}]|^2 (n+q)(n+q-1),
\end{aligned} \tag{33}$$

and $\langle \hat{a}^\dagger \hat{a} \rangle$ is also obtained in (34). From Fig. 2, it is visible that, the inequality $g^{(2)}(0) > 1$ holds for two-mode (linear) charge coherent states with ($f(n) = 1$), as well as the f -deformed charge states with $f(n) = \sqrt{n}$ with respectively charge parameters $q = 1, 3$, which indicate the bunching effect of the corresponding states. Interestingly, the \mathbf{q} -deformation plot shows that it becomes exactly 1, like the canonical coherent states, where $q = 1, \mathbf{q} = 7$. This criteria gets the values less than 1, only for the Penson-Solomon nonlinearity function when we considered charge a $q = -1$, i.e., it shows antibunching (nonclassical) effect in a finite region of $\xi \in \mathbb{R}$. Recalling that this nonclassicality sign becomes 2 for thermal light, we observe from figure 2 that for the

cases $f(n) = \sqrt{n}$ and $f_{PS}(n)$ this function takes values greater than 2 in some regions of ξ , i.e., in view of this criterion the corresponding states behave like a supper-thermal light.

- For the second-order correlation function between the two modes, the following mean values are necessary:

$$\langle \hat{b}^\dagger \hat{b} \rangle^{(+)} = {}^{(+)} \langle \xi, q, f | \hat{b}^\dagger \hat{b} | \xi, q, f \rangle^{(+)} = |c_{q,0}^{(+)}|^2 \sum_{n=0}^{\infty} n |[B_n^{(+)}]|^2, \quad (33)$$

and

$$\begin{aligned} \langle \hat{a}^\dagger \hat{a} \hat{b}^\dagger \hat{b} \rangle^{(+)} &= {}^{(+)} \langle \xi, q, f | \hat{a}^\dagger \hat{a} \hat{b}^\dagger \hat{b} | \xi, q, f \rangle^{(+)} \\ &= |c_{q,0}^{(+)}|^2 \sum_{n=0}^{\infty} |[B_n^{(+)}]|^2 n(n+q), \end{aligned} \quad (33)$$

and $\langle \hat{a}^\dagger \hat{a} \rangle$ is also obtained in (34). Fig. 3 shows $g_{12}^{(2)}(0)$ for two-mode linear and f -deformed charge coherent states. From the figure it is clear that again for the \mathbf{q} -deformation function with charge $q = 2$ and $\mathbf{q} = 7$ this criteria gets exact value 1, like the canonical coherent states; while for all other nonlinearity functions and the chosen charge parameters q the value of $g_{12}^{(2)}(0)$ becomes greater than 1, indicating bunching (classical) effect of the corresponding field.

- For Cauchy-Schwartz inequality, defined in (4.1), the following mean value is necessary:

$$\begin{aligned} \langle \hat{b}^{\dagger 2} \hat{b}^2 \rangle^{(+)} &= {}^{(+)} \langle \xi, q, f | \hat{b}^{\dagger 2} \hat{b}^2 | \xi, q, f \rangle^{(+)} \\ &= |c_{q,0}^{(+)}|^2 \sum_{n=0}^{\infty} |[B_n^{(+)}]|^2 n(n-1), \end{aligned} \quad (33)$$

and $\langle \hat{a}^{\dagger 2} \hat{a}^2 \rangle$ and $\langle \hat{a}^\dagger \hat{a} \hat{b}^\dagger \hat{b} \rangle$ are obtained in (34) and (34), respectively. Fig. 4 shows Cauchy-Schwartz inequality of mode a for two-mode linear and the three types of f -deformed charge coherent states. It is seen from the figure that in all cases this criterion gets negative value for the chosen charge parameters, indicating the nonclassicality feature of the associated states.

- In addition, the probability of finding $n + q$ photons in mode a and n photons in mode b for our introduced positive charge states $|\xi, q, f\rangle^+$ reads as:

$$P^{(+)}(n + q, n) = |\langle n + q, n | \xi, q, f \rangle^{(+)}|^2 = |c_{q,0}^{(+)}|^2 |[B_n^{(+)}]|^2. \quad (34)$$

Fig. 5 shows photon number distribution for all of the considered non-linearity functions including the linear one ($f(n) = 1$). From the figure one may observe that, in all cases the oscillatory behaviour of the photon count is visible for the chosen q and ξ parameters. As is well-known this behaviour is one of the nonclassicality signs of the quantum states. Therefore, in view of this point, all of the considered charge states are nonclassical.

- Quasi-distribution (Husimi) function for the f -deformed charge coherent states with $q \geq 0$ read as

$$\begin{aligned} Q(\alpha_1, \alpha_2, \xi) &= \frac{1}{\pi} |\langle \alpha_1, \alpha_2 | \xi, q, f \rangle^{(+)}|^2 \\ &= e^{-|\alpha_1|^2} e^{-|\alpha_2|^2} \sum_{n=0}^{\infty} \frac{|\alpha_1|^{2(n+q)} |\alpha_2|^{2(n)}}{n!(n+q)!} |c_{q,0}^{(+)}|^2 |[B_n^{(+)}]|^2 \end{aligned} \quad (34)$$

We have plotted the three-dimensional graphs of quasi-probability distribution $Q(\alpha_1, \alpha_2)$ in terms of the amplitudes $x = \text{Re}(\alpha_1)$ and $y = \text{Im}(\alpha_1)$ for various nonlinearity functions including $f(n) = 1$ (linear) with fixed parameters ξ , q and α_2 . Generally, irrespective of the selected nonlinearity functions, the plots corresponding to negative values of q are similar, as is the case for positive q 's. Specifically, from Fig. 6 it is obvious that, in all cases, for the chosen parameters have been used in our numerical calculations, when q gets positive values there exists a hole (centered at the origin of the complex plane) in the three-dimensional graphs, while this hole will be disappeared when the q parameter gets negative values.

We end this section with mentioning the following two points. Firstly, although we brought the necessary mean values for our numerical calculations only for positive charge coherent states $|\xi, q, f\rangle^{(+)}$, the calculation of similar quantities for the $|\xi, q, f\rangle^{(-)}$ states, which have been used in our numerical results may easily be done, too. Secondly, our interpretations on the plotted

graphs specifically concern with the chosen fixed parameters have been indicated in the related figure captions. So, obviously the variation of the related parameters may lead one to enter states with various physical behaviour and different nonclassicality features.

5 Summary and conclusion

In summary, we reobtained two-mode charge coherent states by a standard method rather than $\langle \eta |$ representation. Then, we extended the latter approach to nonlinear coherent states method and introduced the representation of two-mode f -deformed charge coherent states. The construction is valid for a large class of generalized nonlinear oscillators. However, in this paper we only used a few well-known nonlinearity functions associated with particular quantum systems as some physical appearances of our presented formalism. After introducing the explicit form of the associated states in two-mode Fock-space, we established that the f -deformed charge coherent state recovers two-mode charge coherent states when one sets $f(n) = 1$. As a clear fact, it is observed that different $f(n)$'s lead to various classes of deformed charge coherent states, obviously with different physical properties, by tuning the parameters which exist in any case, i.e, ξ , q and sometimes like for the Penson-Solomon and \mathbf{q} -deformation an excess parameter, p and \mathbf{q} , respectively. Anyway, due to the common motivation in the introduction of any generalized coherent states, i.e., investigating the nonclassicality features of the states, we pay attention to this matter by evaluating some of the nonclassicality properties of both two-mode linear and f -deformed charge coherent states. As some criteria, we computed Mandel parameter, second-order correlation function, second-order correlation function between the two modes, Cauchy-Schwartz inequality, probability distribution function in addition to the quasi-probability Husimi function $Q(\alpha_1, \alpha_2)$ of the states, using three types of well-known nonlinearity functions, numerically. Summing up, all of the corresponding states have enough nonclassicality features to be classified in the nonclassical states, i.e., the states with no classical analogue. Although our work is mainly possesses mathematical-physics structure, we mention that the pair coherent states have found a few experimental schemes for their generations, so we hope that the two-mode charge coherent states in (25) and (26) which were originally introduced in [12] and our f -deformed counterpart in (31) and (33) also find their appropriate experimental gener-

ation proposals in near future.

Acknowledgments

The authors would like to thank Dr M Hatami for his very useful helps in preparing computer programs for our numerical calculational results. Also, we thank from the referees for their time and helps which caused to improve and clarify the content of the paper.

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FIGURE CAPTIONS

FIG. 1 The plot of Mandel parameter for mode a (Q_a), as a function of ξ for f -deformed charge coherent states. The dotted line is for $f_{PS}(n)$ with fixed parameter $p = 0.5, q = 1$ and the dashed line is for $f_{\mathbf{q}}$ with fixed parameter $q = 2, \mathbf{q} = 7$.

FIG. 2 The plot of second-order correlation function for mode a ($g_a^2(0)$), as a function of ξ , for f -deformed charge coherent states. The continuous line is for two mode (linear) charge coherent state ($f(n) = 1$) with fixed parameter $q = 1$, dotted line is for $f_{PS}(n)$ with fixed parameter $q = -1, p = 0.5$, dashed line is for $f_{\mathbf{q}}(n)$ with fixed parameter $q = 1, \mathbf{q} = 7$ and the dot-dashed line is for $f(n) = \sqrt{n}$ with fixed parameter $q = 3$.

FIG. 3 The second-order correlation function between two modes ($g_{12}^2(0)$), as a function of ξ , for f -deformed charge coherent states. The continuous line is for two mode (linear) charge coherent state ($f(n) = 1$) with fixed parameter $q = -1$, dotted line is for $f_{PS}(n)$ with fixed parameter $q = -2, p = 0.5$, the dashed line is for $f_{\mathbf{q}}(n)$ with fixed parameter $q = 2, \mathbf{q} = 7$ and the dot-dashed line is for $f(n) = \sqrt{n}$ with fixed parameter $q = 1$.

FIG. 4 The plot of Cauchy-Schwartz inequality (I_0), as a function of ξ , for f -deformed charge coherent states. The continuous line is for two-mode (linear) charge coherent state ($f(n) = 1$) with fixed parameter $q = 1$, dotted line is for $f_{PS}(n)$ with fixed parameter $q = 1, p = 0.5$, the dashed line is for $f_{\mathbf{q}}(n)$ with fixed parameter $q = 3, \mathbf{q} = 7$ and the dot-dashed line is for $f(n) = \sqrt{n}$ with fixed parameter $q = 2$.

FIG. 5 The plot of probability of finding $n + q$ photons in mode a and n photons in mode b in $|\xi, q, f\rangle$, ($P(n+q, n)$), as a function of n , for f -deformed charge coherent states. The continuous line is for two-mode (linear) charge coherent state ($f(n) = 1$) with fixed parameter $\xi = 5$ and $q = 2$, dotted line is for $f_{PS}(n)$ with fixed parameter $\xi = 10$ and $q = -1, p = 0.5$, dashed line is for $f_{\mathbf{q}}(n)$ with fixed parameter $\xi = 5$ and $q = -2, \mathbf{q} = 7$ the dot-dashed line is for $f(n) = \sqrt{n}$ with fixed parameter $\xi = 10$ and $q = 1$.

FIG. 6 The plot of $Q(\alpha_1, \alpha_2)$ as a function of the amplitudes $x = \text{Re}(\alpha_1)$ and $y = \text{Im}(\alpha_1)$ for different classes of states. (a) two-mode (linear) charge

coherent state ($f(n) = 1$), with fixed parameters $\xi = 10$, $q = 1$ and $\alpha_2 = 1 + i$. (b) Two-mode (linear) charge coherent state ($f(n) = 1$), with fixed parameters $\xi = 10$, $q = -1$ and $\alpha_2 = 1 + i$. (c) f -deformed charge coherent state with $f_{PS}(n)$, fixed parameters are $\xi = 10$, $q = 2, p = 0.5$ and $\alpha_2 = 1 + i$. (d) f -deformed charge coherent state with $f_{PS}(n)$, fixed parameters are $\xi = 10$, $q = -2, p = 0.5$ and $\alpha_2 = 1 + i$. (e) f -deformed charge coherent state with $f_{\mathbf{q}}(n)$, fixed parameters are $\xi = 10$, $q = 3, \mathbf{q} = 7$ and $\alpha_2 = 1 + i$. (f) f -deformed charge coherent state with $f_{\mathbf{q}}(n)$, fixed parameters are $\xi = 10$, $q = -3, \mathbf{q} = 7$ and $\alpha_2 = 1 + i$. (g) f -deformed charge coherent state with $f(n) = \sqrt{n}$, fixed parameters are $\xi = 10$, $q = 4$ and $\alpha_2 = 1 + i$. (h) f -deformed charge coherent state with $f(n) = \sqrt{n}$, fixed parameters are $\xi = 10$, $q = -4$ and $\alpha_2 = 1 + i$.